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Probing QCD Corrections to QED Effects Through Optical Experiments in Vacuum

J. Dias de Deus[1)], P. Castelo Ferreira[1)], and J. T. Mendonça[2)]
1) CENTRA, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal and
2) CFP and CFIF, Instituto Superior Técnico, 1049-001 Lisboa, Portugal

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We suggest that the QCD corrections to QED calculations may be probed through optical experiments in vacuum. Formally, the diagram for virtual $e\bar{e}$ production is identical to the one for virtual $q\bar{q}$ production. However due to confinement, or to the growth of α_s as p^2 decreases, a direct calculation of the diagram is not allowed. At large p^2 we consider the virtual $q\bar{q}$ production diagram, in the intermediate region we discuss the role of the contribution of quark condensates $\langle q\bar{q}\rangle$ and at the low-energy limit we consider the resonance of the π^0. We conclude that the π^0 resonance dominates. To achieve such measurements it will be necessary to significantly increase the present experimental accuracy. It would then be possible to test the existence of quark condensates and respective cut-off energy.

In a recent work [1], we have shown that a rotating magnetic field in vacuum can excite several sidebands, therefore explaining the recent experiments by Zavattini et al. [2]. This could be a clear observation of the Schwinger-Euler-Heisenberg predictions for vacuum polarization due to electron-positron virtual loops [3] have been observed experimentally [1, 2]. Optical experiments seem today one of the best candidates to probe low energy physics and optical properties of vacuum including photon-splitting [4, 5], photon-photon interactions [6, 7], or the possible detection of pseudo-scalar particles [8], pseudo-photons [9], paraphotons [10] and millicharge fermions [11]. Here we address the possible perturbative effects due to the strong interactions. Namely we analyze quark loops, quark condensates and meson contributions to vacuum polarization.

The polarization of the vacuum due to electron-positron virtual pair production is a well known phenomenon. Naively, we can expect that the same kind of physics applies to quark-antiquark virtual pair production. In the presence of an external field we have in general the diagram of figure 1. We write in the case of $q\bar{q}$ virtual pair production for the order of α^2 the Euler-Heisenberg Lagrangian [3, 4, 5]

$$\begin{aligned}\mathcal{L}^{(2)}_{q\bar{q}} &= \xi_q\left[4\left(F_{\mu\nu}F^{\mu\nu}\right)^2+7\left(\epsilon^{\mu\nu\delta\rho}F_{\mu\nu}F_{\delta\rho}\right)^2\right]\ ,\\ \xi_q &= \delta_q\frac{2\alpha Q_q^2}{45\,(B_c^q)^2}\ ,\\ B_c^q &= 3\frac{m_q^2c^2}{e\,Q_q\,\hbar}\ .\end{aligned} \tag{1}$$

Here the factor of 3 comes from the summation over colors and Q_q stands for the quark fractional charge. As for the quark masses m_q correspond to the *renormalized* masses that appear in the quark propagator. The relation of the polarization due to $q\bar{q}$ with the polarization due to $e\bar{e}$ corresponding to electron-positron loops is

$$\begin{aligned}\Pi^{(2)}_q &= \Delta\xi_q\,\Pi^{(2)}_{e\bar{e}}\ ,\\ \Delta\xi_q &= \frac{\xi_q}{\xi_e}=3\,\delta_q\left(\frac{m_e\,Q_q}{m_q}\right)^4\ .\end{aligned} \tag{2}$$

Here $\delta_q = w_{\Lambda_q}/w_{\rm tot} < 1$ is a phase space correction due to confinement of strong interactions. We know that at low energies there are no free quarks, therefore quark loops carrying small momenta cannot be considered. The way out is to introduce a lower cut-off Λ_q in the loop momenta such that only the high momenta contribution to the loop is considered.

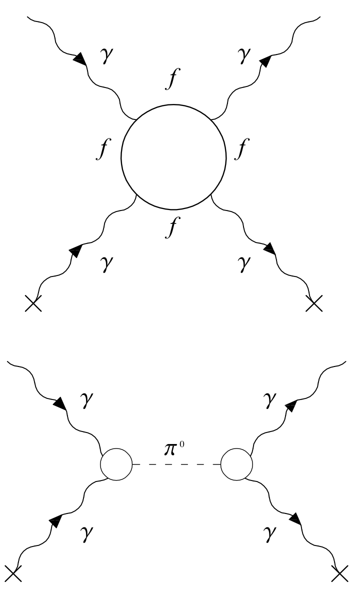

FIG. 1: *The diagrams for fermion-antifermion loops and the exchange of a* π^0 *neutral meson. The vertex* $\pi^0\gamma\gamma$ *includes the axial anomaly.*

The probability for the full range of momenta (i.e. $p^2 \in]0, +\infty[$) is given by the series [12]

$$w_{\rm tot} = \frac{\alpha\,\mathbf{B}^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\frac{n\pi m_q^2}{|Q_q \mathbf{B}|}} \ . \tag{3}$$

Due to confinement and the increase of α_s for small values of p^2, we introduce a cut-off Λ_q that truncates the series (3) by excluding the low p^2 region.

$$w_{\Lambda_q} = \frac{\alpha\,\mathbf{B}^2}{\pi^2} \sum_{n=n_{\Lambda_q}}^{\infty} \frac{1}{n^2} e^{-\frac{n\pi m_q^2}{|Q_q \mathbf{B}|}} \ . \tag{4}$$

We obtain the relation

$$n_{\Lambda_q} = \left(\frac{\Lambda_q}{m_q}\right)^2 \ . \tag{5}$$

For the light quarks with mass of order $m_q \sim 10\,MeV$ [13] we have that $n_{\Lambda_q} \sim 3600$ holding $\delta_q \sim 10^{-10^{12}}$. Here we considered $\Lambda_q \sim 600\,MeV$, this is the value for which the strong interactions coupling constant becomes unity $\alpha_s \sim 1$ [14] such that below this energy threshold, QCD is in a non-perturbative regime. The free quark loop contribution to vacuum polarization is therefore negligible. This contribution will only be relevant for very strong magnetic fields of order $B \sim 10^{12}\,T$ which is only achievable near neutron stars and magnetars [15].

The only well established low-energy resonance quark state (corresponding to the light mesons) are the π's. In low energy physics these particles can be used as *fundamental* bosons within the framework of Chiral Perturbation Theory (ChPT) [16]. Therefore below the cut-off $p^2 < \Lambda_q$ the main contribution is due to this meson

$$\begin{aligned} \Pi^{(2)}_{\pi^0} &= \Delta\xi_{\pi^0}\,\Pi^{(2)}_e \ , \\ \Delta\xi_{\pi^0} &= \frac{\left|\mathcal{M}(2\gamma \to \pi^0 \to 2\gamma)\right|}{\left|\mathcal{M}(2\gamma \to 4e \to 2\gamma)\right|} \\ &= \frac{180\, m_e^4}{\pi^2\, m_{\pi^0}^2\, f_\pi^2} \ . \end{aligned} \tag{6}$$

For higher masses the contributions are of lower magnitude, therefore the other meson effects are negligible when compared to the π^0 effect. There is however another not so well established contribution that we can consider. In the presence of a background magnetic field there is a vacuum polarization contribution due to quark condensation. Here we will use the results obtained using Schwinger-Euler-Heisenberg formalism [3] in the context of ChPT [17]

$$\begin{aligned} \Pi^{(2)}_c &= \Delta\xi_c\,\Pi^{(2)}_e \ , \\ \Delta\xi_c &= \frac{\xi_q}{\xi_e} = \frac{90 m_e^4}{96 f_\pi^4} \ln\left(\frac{\Lambda^2}{m_\pi^2}\right) \ . \end{aligned} \tag{7}$$

Here Λ stands for the ultra-violet cut-off for the quark condensate, m_π for the pion mass is the infrared cut-off and f_π to the pion form factor. We follow by giving some details on how quark condensates are obtained and explain which regimes exist depending on the loop momentum. The vacuum polarization for ChPT is given by the integral

$$\begin{aligned} \Pi_{\langle q\bar q\rangle} &= \int_0^\infty ds\, I_{\langle q\bar q\rangle} \ , \\ I_{\langle q\bar q\rangle} &= -\frac{\alpha\,B}{12\, f_\pi^4} \frac{1}{s^2} \left[\alpha\,B\,\cot(\alpha\,B\,s) - \frac{1}{s}\right] \ . \end{aligned} \tag{8}$$

This distribution is represented in figure 2.

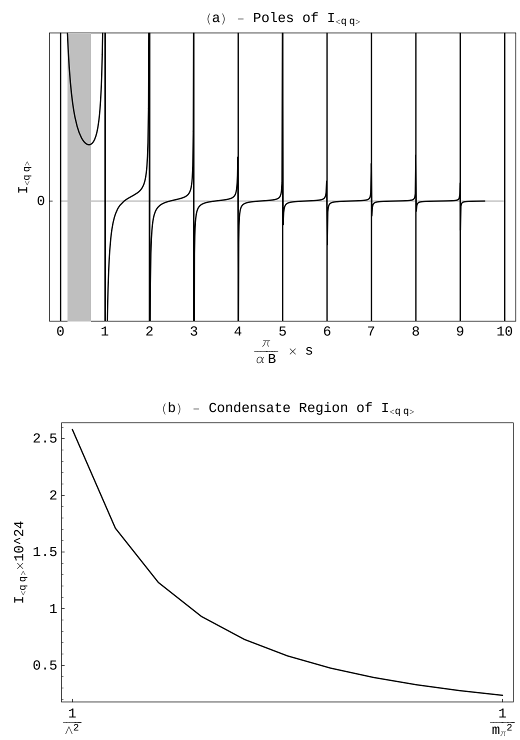

FIG. 2: *(a) The integrand (8). The poles at $s = (n-1)\pi/\alpha B$ (for $n = 1, 2, \ldots, \infty$) are market by vertical lines and contribute to the pion vacuum polarization. (b) The same integrand between both cut-offs $m_\pi = 135\,MeV$ and $\Lambda = 300\,MeV$ for $B = 5.5\,T$. It corresponds to the marked region between the poles at $s = 0$ and $s = \pi/\alpha B$ of (a).*

The contributions that should be consider are due to the poles below the cut-off $s < 1/\Lambda^2$. For

p (MeV)	$\Delta\xi_{\pi^0}$	$\Delta\xi_c$	$\Delta\xi_q$
> 600	0	0	$10^{(-10^{12})}$
$140 - 600$	0	1.32×10^{-9}	0
< 140	7.83×10^{-9}	0	0

TABLE I: The several QCD effects and their magnitude for the several ranges of the pair momenta p.

weak fields the only pole that contributes for pion loops is at $s = 0$. It corresponds to the $\pi^+\pi^-$ loops and the relative magnitude of its effect is $\Delta\xi_{\pi^+\pi^-} = (m_e f_\pi/2m_\pi^2)^4/2$. We recall that above the cut-off $s > 1/\Lambda^2$ we should consider the quark loops instead of the meson distribution. The novel interesting feature is that we have a new contribution between the pole $s = 0$ and $s = \pi/\alpha|B|$ that corresponds to the quark condensate. We note that from a more fundamental level based in Nambu-Jona-Lasinio theory [18] quark condensates holding the same order of magnitude for the quark condensate value have been obtain in [19]. There is an important point to stress, NJL consider explicit actions for the quarks instead of the effective actions for the mesons considered in ChPT, the condensate cut-off Λ should correspond in NJL to the confinement energy. These theories were originally motivated by superconductivity and the relation between ChPT and NJL is equivalent to the relation between Landau-Ginzburg effective theory [20] and the Bardeen-Cooper-Schriffer microscopic theory [21] for superconductivity.
For $m_e = 0.51\,MeV$, $f_\pi = 93\,MeV$, $eB \approx 4.7 \times 10^{-7} MeV$ ($B \approx 5.5T$) and $\Lambda \approx 300\,MeV$ we obtain the relative magnitude of the π^0 contribution as given by (6)

$$\Delta\xi_{\pi^0} \approx 7.83 \times 10^{-9} \ , \tag{9}$$

and for the condensate as given by (7)

$$\Delta\xi_c \approx 1.32 \times 10^{-9} \ . \tag{10}$$

The next contribution (concerning magnitude) is from pure QED and corresponds to the muon-antimuon loop holding a relative magnitude of $\Delta\xi_\mu \approx 5.43 \times 10^{-10}$. The contribution from the pion loop holds a lower magnitude, $\Delta\xi_{\pi^+\pi^-} \approx 1.43 \times 10^{-12}$. We list the allowed effects and their magnitude for several ranges of p in table I.
The relevant radiative corrections to the usual classical wave equation in order α^2 is linear in the photon field A [5]. For a static transverse magnetic field B_0 we have the two eigenvalues [4, 5]

$$\begin{aligned} \lambda_\parallel &= 14(1 + \Delta\xi_{QCD} + \Delta\xi_{QED})\,\kappa\, B_0^2 \ , \\ \lambda_\perp &= 8\,(1 + \Delta\xi_{QCD} + \Delta\xi_{QED})\,\kappa\, B_0^2 \ . \end{aligned} \tag{11}$$

The directions $\parallel$ and $\perp$ correspond respectively to the parallel and transverse directions to the external magnetic field and $\kappa = \frac{2\alpha^2\hbar^3}{45 m_e^4 c^5} = 2.1 \times 10^{-21}\, T^{-2}$. We thus have

$$\Delta\xi_{QCD} = \Delta\xi_{\pi^0} + \Delta\xi_c + \Delta\xi_{\pi^+\pi^-} + \Delta\xi_q + \ldots \ , \tag{12}$$

where the dots represent other lower magnitude contributions, such as the ones from the other mesons. For the order of magnitudes considered here the only relevant correction due to pure QED is from the muon loops such that $\Delta\xi_{QED} = \Delta\xi_\mu + \ldots$, we stress however that the dominant contribution is from the π^0.
The above equations result in having different refractive indices in the parallel and perpendicular directions to the magnetic field [5]

$$N_\parallel = 1 + \frac{1}{2}\lambda_\parallel \ \ , \qquad\qquad N_\perp = 1 + \frac{1}{2}\lambda_\perp \ , \tag{13}$$

which introduce a phase shift in the propagating wave. A linearly polarized wave of wave number $\mathbf{k} = k_0\,\mathbf{z}$ which polarization makes an angle of θ_0 with a static magnetic field gains an ellipticity due to the relative phase shift between both propagating modes $\Delta\phi = (N_\parallel - N_\perp)\Delta z$, being Δz the distance traveled by the light. The effective rotation of the polarization is

$$\varphi_i = \theta_0 - \arctan\left[\frac{\cos\left(\lambda_\perp\, k_0\, \Delta z/2\right)}{\cos\left(\lambda_\parallel\, k_0\, \Delta z/2\right)}\right] \ . \tag{14}$$

The question that remains is how can we measure the effects of QCD? The present measurable angular rotation of polarization is $10^{-9}\,rad$ [2] which clearly is not enough.
We can increase the overall polarization rotation by a multiplicative factor of 10^6–10^{12} by increasing the optical path by a multiplicative factor of 10^3–10^6. This can be done by:

1. increase the interaction length by a multiplicative factor of 10–10^3;

2. decrease the wave length by a multiplicative factor of 10^2–10^3,

together with the significant improvement on the signal to noise ratio by a multiplicative factor of 10^2–10^3, would allow to measure the rotation with enough accuracy to identify the contributions due to the QCD effects presented here. The relative

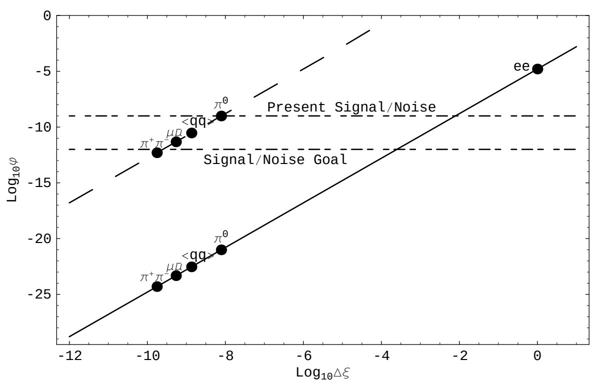

FIG. 3: *Contributions to the polarization rotation from the several effects as a function of $\Delta\xi_i$, i.e. each effect magnitude in relation to the magnitude effect due to electron-positron loops ($e\bar{e}$). Both axis are in logarithmic scale. The continuous line coincides with the PVLA experimental conditions [2]. The marked points are labeled and correspond to the QED corrections due to electron-positron loops ($e\bar{e}$), interchange of the neutral pion (π^0), quark condensates ($\langle q\bar{q}\rangle$), the muon-antimuon loops ($\mu\bar{\mu}$) and charged pion loop ($\pi^+\pi^-$). The dashed line corresponds to an increase of $\Delta z \to 10^3 \Delta z$ and $k_0 \to 10^3\, k_0$ in relation to the continuous line. The present day signal to noise ratio and the signal to noise ratio goal are also plotted.*

magnitude of the several effects versus the contribution to the polarization rotation for present day state of the art and considering the improvements just proposed are illustrated in figure 3.

Achieving the above goals may be a challenging task but it may be worthwhile. If QCD effects are measured we would be testing the existence of quark condensates due to the magnetic background field and have an accurate estimative of the condensate ultra-violet cut-off Λ. We note that the usual QCD scale is set by $\Lambda_{QCD} \approx 200\, MeV$, however as already discussed, we also know that for energies of approximately $\Lambda_q = 600\, MeV$ the strong running coupling constant α_s is of order of unity and the perturbative regime of QCD is no longer valid [14]. Therefore the correct value of the cut-off corresponding to low-energy quark condensate is not exactly known and should be in the range $200 < \Lambda < 600\, MeV$. This value should correspond to the chiral phase transition energy of the Nambu-Jona-Lasinio theory [18]. Although the critical energy that corresponds to the chiral phase transition may be measured, the mechanisms of confinement and chiral symmetry breaking cannot be accessed directly by this sort of experiments due to the very weak available magnetic fields. As already mentioned, eventual effects of chiral symmetry breaking and real pair production, would only be observable above the critical magnetic fields, i.e. $B \geq 10^{12}\, T$. These values can only be accessible near neutron stars and magnetars [15].

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